The Natural Emergence of Category Effects on Rugged Landscapes

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Abstract

Category theory finds that markets partition producers into categories and that producers who do not fit one specific category—or who span multiple categories—perform worse than their single-category peers. The dominant thread of this work argues that this miscategorization penalty arises when cognitive limits of categorization cause individual members of the market's audience to exclude or denigrate ill-fitting producers. I present a null model of markets in which a miscategorization penalty appears without being caused by a market audience: Drawing on cognitive science and research on rugged landscapes, the model shows that producer herding behavior generates a spurious correlation between market outcomes and miscategorizations. The model further predicts the dynamics of categorical emergence and change over time. I establish these results in a simulation and discuss strategies by which this landscape model can be empirically distinguished or integrated with the cognitive account.

1 Introduction

Category theory combines a theoretical proposition with an empirical finding. The theory describes how individual cognition processes induce market conformity (Zuckerman, 2017; Zhao et al., 2017): Markets are full of ambiguous objects for audiences to sort through. People simplify search by categorizing objects. They struggle to understand, evaluate, or consume objects that are hard to categorize (Zuckerman, 1999; Hannan et al., 2007, 2019). Evidence that people categorize is well-established in cognitive psychology (Rosch et al., 1976; Goldstone, 1994; Murphy, 2004) and marketing (Shocker et al., 1991; Roberts and Lattin, 1991). Category theory deploys this individual process to explain an empirical finding: markets routinely assign categorical labels to objects, and objects with multiple or ambiguous labels face penalties. Restaurants specializing in a single cuisine (Korean or Mexican) earn higher reviews than those serving mixed cuisines (Korean-Mexican, Kovács and Hannan, 2015). Audiences prefer single-genre (romance or horror) to multi-genre movies (romance-horror, Hsu, 2006). The path from theory to finding seems clear: Miscategorized objects fail in the market because individuals find it hard to understand them. A stream of research measuring the penalty to miscategorized—poorly- and multi-labelled—firms appears to support the story (Hannan, 2010; Durand and Paolella, 2013; Vergne and Wry, 2014).

But various findings complicate the attempt to add up individual categorizations into a market-scale penalty. The penalty itself sometimes reverses (Smith, 2011; Leung, 2014; Sgourev and Althuizen, 2014; Paolella and Durand, 2016). Markets harbor multiple audiences with differing tastes (Kovács and Liu, 2016) and differing preferences for categorical ambiguity (Pontikes, 2012; Goldberg et al., 2016). Further still, work in cognitive psychology suggests that individuals are categorically flexible, capable of integrating multiple categorical systems (Medin et al., 1997), devising novel categories on the fly (Barsalou, 1983; Durand and Paolella, 2013), and even ignoring categories when they do not clearly apply to the problem at hand (Proffitt et al., 2000). The categories themselves can shift, and markets can appear and disappear (Christensen and Bower, 1996; Durand and Khaire, 2016). Audiences do not appear strongly bound by their categorical schemes. Producers will struggle to identify which standard they must conform to.

This paper proposes a null model (Starbuck, 1994; Gotelli and Graves, 1996) that accounts for the findings of category theory by focusing on producer decisions at the market interface. Null models generalize the idea of a null hypothesis, showing that some theorized causal relationship may arise as a statistical consequence of simple assumptions that exclude the causal mechanism.[1] This paper relies on a set of minimal assumptions combining two streams of work outside of category theory: First, work in psychology and anthropology has recognized that real-world categories tend to align to natural divisions of the environment (Malt, 1995; Brown, 2004) and that independent cultures tend to group similar sets of objects together. Second, a literature on strategic positioning in markets has identified conditions under which producers agglomerate in the marketplace (Hotelling, 1929; Lancaster, 1990), and when such agglomerations result from the natural heterogeneity within markets and audiences (Levinthal, 1997; Adner et al., 2014; Lenox et al., 2007)—from the ruggedness of market landscapes.

In the present model, producers struggle to find optimal positions in a rugged market landscape, balancing competition from peers against uncertainty around the location of consumer niches, categorical expectations, or the viability of specific production processes (March et al., 1991; Hannan et al., 2003). These forces herd producers into dense clusters formed around successful positions identified by prior entrants. In parallel, a passive audience labels these naturally formed agglomerations, enacting a means of describing the market without affecting outcomes within it.

This landscape model replicates the static findings of category theory, predicts categorical emergence and dynamics, and identifies several boundary conditions on the existing theory of market categories. In each case, the paper identifies the limits of this null model and highlights steps that a cognitive theory of categories must take to establish its own predictions in any particular market. The paper discusses the theoretical issues at play, and then describes the model and its results. It concludes with strategies for comparing the landscape and cognitive models and for integrating social and material constraints into a richer categorical theory of markets.

[1] While the null model terminology arises from ecology (Connor and Simberloff, 1983, 1979), similar approaches have a long history in social sciences: the preferential attachment model (Barabasi and Albert, 1999), Levinthal's "baseline model" of organizational mortality (1991), Schelling's tipping point mechanisms (1971), or Simon and Bonini's stochastic model of firm size (1958) all aim to provide a parsimonious stochastic alternative to prior explanations of some phenomenon.

2 Theoretical Background

2.1 Constructs in Category Theory

To establish the relationship of this article to category theory, it is helpful to examine the role of three constructs in the theory. First, objects in a market have a *position*, denoting the characteristics of an object that determine how it might appeal to various needs. Second, objects receive a *categorization*, the process by which audience members perceive the object's characteristics and communicate those to one another. Third, objects experience an *outcome*, some measurable degree of success in the market, whether through audience appeal, evaluations, prices, or sales.

The dominant audience-driven perspective in category argues that objects' categorizations mediate the link from their positions to their outcomes: Audiences categorize objects based on their characteristics, but the categorization process determines whether these objects are successful in the market (see Fig. 1a). The seminal works of Zuckerman (1999) and Hannan et al. (2007) set up this argument: Zuckerman (1999) describes how audience categorizations operate in a sorting process that selects against miscategorized objects, establishing the second link of fig. 1a. Hannan et al. (2007; further elaborated in Hannan et al., 2019) consider objects in an abstract feature space—their position—and describe how audiences assign categorical labels to clusters of similar objects in the space. This establishes how subsequent objects become categorized, the second link of fig. 1a.

[Figure 1 about here.]

Empirical work in category theory relies on the structure provided by these core pieces. The feature space model of Hannan et al. allows for increasingly sophisticated measures of producer categorization (e.g. Pontikes, 2012; Pontikes and Hannan, 2014; Kovács and Hannan, 2015; Hannan et al., 2019). Zuckerman (1999) establishes the plausible causal link from these categorizations to observable outcomes. Empirical work proceeds by showing an association between market-level measures of categorization and outcomes.

2.2 Categorization and Labels

The explanatory leap from positions to outcomes via categorizations obscures a theoretically critical aggregation. The cognitive foundation of category theory rests on an argument that individual categorization processes exhibit predictable imperfections. Indeed, work in category theory that examines individual categorizations supports this leg of the argument (Negro and Leung, 2013). The theory then proposes that individual imperfections aggregate into collective flaws, that 'market categories' are the sum of many individual categories.

There are several concerns with this position. First, individuals appear to be categorically flexible. People form *ad hoc* categories to resolve unexpected or infrequent situations (Barsalou, 1983; Durand and Paolella, 2013). People use distinct category systems to classify the same objects, selecting the best categorizations for a given task (Medin et al., 1997), and incorporating non-categorical causal arguments to make predictions about specific objects (Proffitt et al., 2000). Second, collective categorizations do not simply aid individual categorization but serve collective coordination and communication functions. Collective concepts are constrained by the need for efficient, expressive, and socially conformant communication (Goldberg, 2019). Situation-specific languages emerge spontaneously (Weber and Camerer, 2003) and converge on mutually comprehensible terms (Guilbeault et al., 2021). In short, individuals appear to be able to flexibly adapt their categorical schemata to the needs of specific situations.

At the same time, the stability of market categories appears to reflect the general tendency of cognitive concepts to anchor on material constraints. Despite the flexibility of individual categorizations, categories in the wild obey various regularities (Murphy, 2004). Categories bind objects with some degree of internal similarity and external distinction (Rosch and Mervis, 1975; Goldstone, 1994; Goldstone and Son, 2012). Hannan et al. (2007) likewise argue that categories are most meaningful when they refer to dense, distinct clusters of similar objects. In addition, people tend to rely on a basic level of categorization for any particular object—referring to cats as 'cats,' for example, and not by specific breeds or as 'animals' (Rosch et al., 1976; Murphy, 2004). Across cultures, societies tend to align on similar categorizations of specific objects (Malt, 1995; Brown, 2004; Xu et al., 2013), such as different kinds of plants or animals; in the case of biological categorizations, folk categories frequently map onto scientific taxa. In general, collectively held categories appear to frequently reflect natural kinds—natural clusters and distinctions. There is little reason to suspect that market categories follow some different principle.

A finding that collectively held categorizations predict product outcomes theoretically implicates individual categorization processes only if no other causal mechanism could produce collective categorizations. This article lays out a mechanism by which natural kinds emerge in the market.[2]

2.3 Agglomeration, Environmental Complexity, and Imitation

There is an extensive economic literature on positioning, product variety, and agglomeration (Hotelling, 1929; Biscaia and Mota, 2013; Fujita and Thisse, 1996; Lancaster, 1990). Such work tries to explain how producers position their products and the circumstances under which they will tend to cluster or separate from each other. For this paper, a key distinction among classes of mechanisms involves assumptions about variation in the product 'landscape'—whether there are positions of high or low appeal, customer density, or other local complications. A large class of papers identify mechanisms that generate clustering in the absence of exogenous variation, identifying various strategic implications to agglomeration or efficiencies that emerge as producers collocate. Such research aims to show that agglomerations can emerge even under strong assumptions about the environment—that is, even in homogeneous environments in which no position is favored *ex-ante*. Such models characterize an important set of mechanisms, but they are often sensitive to specific behavioral and environmental assumptions (see e.g. Salop, 1979; D'Aspremont et al., 1979).

This article instead assumes that exogenous environmental variation provides a more general description of the world, following a tradition of organizational work on environmental and organizational complexity, and on rugged landscapes. Externally, organizations are subject to environmental opacity and complexity, resulting in unanticipated outcomes across both time (Levinthal, 1991) and position (Levinthal, 1997). They only perceive limited information about their competitors (White, 1981). Internally, organizations have a limited understanding of their own routines and production processes (Nelson and Winter, 1982; Hannan et al., 2003; Bernstein, 2012). Attempts to achieve a particular market position may be hampered by unobservable components of a strategy (Rivkin, 2000) or by the impossibility of generating critical resources (Wernerfelt, 1984; Teece et al., 1997). In general, producers' ability to predict the success of a given position in the market may be very limited.

In such complex environments, producers' positioning is substantially driven by informational

[2] It must be pointed out that social classification is itself subject to interpretation as a strategic producer decision. Signaling is a well-established market dynamic (Spence, 1973), and producers will adopt labels that aid their performance in the market (Podolny, 2008; Etzion, 2014; Pontikes and Kim, 2017). This paper discusses the emergence of classification schemes in the absence of any market benefit to classification.

availability. Producers entering the marketplace must balance competitive pressure against the informational advantages of close imitation of competitors. In attempting to maximize their outcomes, producers will effectively face a choice between the imitation of successful predecessors and exploratory differentiation, producing a population that is at once exploratory and agglomerative (cf. Banerjee, 1992; Strang and Macy, 2001; Denrell and March, 2001; Denrell and Le Mens, 2007). As even efforts to conduct market research or pursue lean strategies (Ries, 2011) can at best mitigate but not remove uncertainty, such agglomerative pressures will operate in complex markets. Distinction always bears the threat of failure, which will deter organizations hoping to escape competition.

2.4 A Landscape Model of Category Formation

This article asserts that a rugged landscape model (Levinthal, 1997) of markets contains the minimal elements necessary to produce a marketplace featuring product agglomerations. These agglomerations act as natural kinds to anchor the formation of collective categorizations. Collective categorizations do not affect producers' outcomes in that market. To the extent that miscategorization effects appear within such markets, those effects are epiphenomenal to producers' herding behavior in a rugged landscape (fig. 1b).

I examine three questions about this model: first, whether it reproduces the basic patterns of category theory; second, whether it accounts for the problem of categorical dynamics and emergence; and third, whether it generates distinguishing predictions from the cognitive account.

2.4.1 Question 1: Reproduction of the category penalty

Category theory predicts that markets punish miscategorized objects that cannot enter audience members' consideration or that violate categorical rules. Empirically, this manifests in the finding that category spanners—objects that are classified into multiple categories—suffer relative to single-category objects. Conceptually, this argument also covers objects that fail to attract any category label. Under the landscape model, this penalty arises from producers' tendency to cluster around positions of proven success.

2.4.2 Question 2: Category dynamics and emergence

Category theory typically considers markets in categorical equilibrium and relegates the problems of categorical emergence and change to ancillary mechanisms. Research has emphasized that producers and audience members rely on strategic action to shift categories. Some researchers discuss the role of social movements in creating new categories (Lee et al., 2017; Weber et al., 2008; Carroll and Swaminathan, 2000), or the strategic actions of organizations themselves (Kennedy, 2008; Pontikes and Kim, 2017; Grodal, 2018). Others discuss the role of influential vanguards (Koçak et al., 2014; Rao et al., 2005; Ruef, 2000). Scholars are generally pursuing mechanisms of category emergence (Glynn and Navis, 2013; Durand and Khaire, 2016; Lo et al., 2020). Another line of work discusses how categories shift over time, considering the role of successful exemplars (Zhao et al., 2018), of strategic manipulation of categorical boundaries by incumbents (Hsu and Grodal, 2015), or of categorical violations (Rao et al., 2005). These approaches emphasize the intentionality behind category emergence and stress categorical definition as a prerequisite for markets.

In the landscape model, producers react to each other and explore the landscape over time, and collective categorizations evolve alongside them. Thus, the landscape model directly examines how categories shift, emerge, and disappear over time. It accounts for both stable and shifting markets using a single set of mechanisms. Moreover, it establishes conditions under which it is possible to predict categorical change and emergence.

2.4.3 Question 3: Distinguishing landscape effects from audience cognition

Finally, this article aims not only to provide an alternative mechanism for categorization (fig. 1a vs fig. 1b), but also to provide some guide for how the two models might be empirically distinguished or integrated (fig. 1c). In general, distinguishing social constructionist from structuralist accounts of categorization is difficult (cf. Malt, 1995). Nevertheless, this article explores several avenues, each relying on the ability of a formal model to provide deep insight into the mechanisms that drive specific outcomes. First, the model characterizes the core findings of a spanning penalty (Question 1) as a stochastic outcome with a predictable probability of reversal. Second, the model shows how informational shocks cause predictable shifts in categorical structure. And third, by exploring the space of possible competition structures, the model is able to identify boundary conditions on markets under which categorical effects should be expected to disappear or reverse.

3 Model

This paper models the behavior of entrepreneurial producers sequentially entering a rugged landscape and observing the outcomes of their predecessors. In parallel, a passive audience observes and applies categorical labels to the producers in the market. I discuss the landscape model, the producer entry decision, and finally the audience categorization process.[3]

3.1 Rugged Landscapes and Brownian Paths

Research on rugged landscapes has typically relied on NK landscapes to model performance environments (Levinthal, 1997; Kauffman and Weinberger, 1989) and organizational positioning problems (Lenox et al., 2007; Adner et al., 2014). This paper instead models a rugged landscape as a Brownian path, a novel machinery for examining complex environments (Callander, 2011; Ganz, 2020; Callander and Matouschek, 2019). Unlike NK models, this approach allows producers to positions themselves continuously, and it simplifies the calculation of producers' optimal entry points.[4]

In this model, the fitness landscape is defined by an implicit Brownian path. Following the Hotelling tradition, producers locate themselves continuously along the real number line, with each point representing some market position (Hotelling, 1929; Salop, 1979). The landscape is unbounded, with producers able to locate near or far from each other. Positions that are close to one another represent similar products, products that act as close substitutes in the market. The outcome assigned to a given position—that is to say, the inherent appeal or quality of a given position—is given by the height of a Brownian path at that point. Denoting a market position as x, its appeal is A_x. The basic property of the Brownian path is that the difference in appeal between two positions is given by a normal distribution with variance proportional to the distance between the positions

[3] Code implementing this model is available in an online repository (https://anonymous.4open.science/r/os-categories-42D7).

[4] Other alternatives exist. Multi-arm bandit models suffer from continuous positioning issues. PN landscapes, as recently proposed by Rahmandad (2019), like NK landscapes, complicate the calculation of optimal search behavior given bounded knowledge of the environment. Brownian landscapes are closely related to the family of rugged landscape models. They can be shown to be a mathematical limit of infinite-N, low-K NK landscapes (cf. Weinberger, 1991). In addition, they can be interpreted as a spatially embedded, continuous bandit model.

and a possible directional drift term μ:

$$A_x - A_y \sim N((x-y)\cdot\mu, |x-y|\cdot\sigma^2)$$

I set $\mu = 0$ to model a homogeneous landscape where any position is *ex-ante* equivalent. Other work on Brownian landscapes considers $\mu \neq 0$ to explore the efficiency of search (e.g. Callander, 2011).[5] In addition, because σ^2 acts only as an arbitrary scale on landscape distances, I set $\sigma^2 = 1$ for convenience. This reduces the landscape's governing equation to:

$$A_x - A_y \sim N(0, |x-y|) \tag{1}$$

The appeal function A encapsulates and abstracts the producer's costs and the heterogeneous tastes of consumers at a given position, representing the inherent appeal of a given position to the producer. Nearby producers experience similar production issues and similar desirability to the audience; distant producers experience unpredictable differences on both dimensions due to the complexity of the market.

In practice, because this landscape exhibits infinitesimal variation, fitness values are sampled only when producers enter at a particular point.[6] In effect, while there is a latent landscape within each market, it is unobservable to producers and audiences, as well as to the modeler. Finally, because the landscape is defined only through differences from prior points, I anchor the landscape by initially fixing the appeal at the origin, $A_0 = 0$.

3.2 Producer Competition and Entry

Producers enter the landscape one at a time over multiple periods, attempting to maximize their outcomes given both uncertainty and competition from prior entrants. An initial entrant enters at the origin as an anchor for the landscape. As producers enter the market in each subsequent

[5] Other organizational research has used Brownian paths to model organizational buffers against failure (Levinthal, 1991; Denrell, 2004; Le Mens et al., 2011). While the mechanics of Brownian paths are the same, the Brownian path here represents an unrelated construct.

[6] Briefly, a position at the extreme left or right of the landscape with a closest neighbor at distance Δ (fitness A_C) draws fitness from the normal distribution $N(A_C, \delta)$. A position between two known points at $x_l < x_r$ (fitness A_l, A_r) draws fitness according to a Brownian bridge, drawing from the normal distribution $N\left(A_l + \Delta\frac{A_r - A_l}{x_r - x_l}, \frac{\Delta(x_r - x_l - \Delta)}{x_r - x_l}\right)$. See Appendix 1 for further detail.

period, we can define the set $\mathcal{X}_t$ of producers' position in the market at period t. These positions are publicly known, as are the appeals of each position. The appeal of each position is drawn according to the distribution of the Brownian walk at the point that a producer enters at that position.

Competitive pressure takes the form of a simple penalty to the appeal of a particular position depending on its distance to its nearest neighbors on the left and right. A producer considering market entry in period t at position x would consider their immediate neighbors to the left x_l ($x_l = \max_{x>y\in\mathcal{X}_t} y$) and right x_r (analogous). The competitive penalty is then $c(x, \mathcal{X}_t) = \frac{1}{x-x_l} + \frac{1}{x_r-x}$. If there is no competitor on a given side, then that component of the penalty is set to 0; if x_r does not exist, for example, the penalty is $c(x, \mathcal{X}_t) = \frac{1}{x-x_l}$. Fig. 2 illustrates the penalty. If the producer enters at that position, they would then observe its appeal A_x, and the producer's market outcome will be $A_x - c(x, \mathcal{X}_t)$. This penalty offers a greatly simplified version of price- or patent-based competition and is chosen to simplify analysis of the model. In particular, because the penalty approaches infinity at perfect imitation, it ensures that no two producers locate at the same market position. Appendix 2 examines the multinomial logit model of product choice (McFadden, 1974, 1984; de Palma et al., 1985; De Palma et al., 1987) and finds a similar pattern of results when producers face more realistic competition and are able to collocate.

[Figure 2 about here.]

Producers enter at the position that maximizes their expected utility $u(A_x - c(x, \mathcal{X}_t))$. The choice of utility function is primarily determined by the behavior of expected outcomes on the extremes of the landscape. In period t, if x_R is the rightmost producer in the market ($x_R \geq x$ for all $x \in \mathcal{X}_t$), then $E(A_x) = A_{x_R}$ for all $x > x_R$: any position beyond the most extreme producer has the same expected appeal. Because competitive pressure decreases with distance from the incumbent, risk-neutral producers prefer to locate infinitely far away from the most extreme producers. It is possible to resolve this concern by limiting the span of the environment (e.g., by adopting a circular landscape, as in Salop, 1979) or by limiting the search distance (e.g. through heuristic search rules avoiding extreme positions, or by imposing a search cost increasing in distance). Instead, I assume that producers are risk-averse, and avoid entering at extreme positions due to fear of the uncertainty involved. In particular, u must exhibit decreasing absolute risk aversion (Pratt, 1964; Callander and Matouschek, 2019), and I adopt the specific utility function $u(y) = ay - \exp(-by)$, with $b > 0$.

Appendix 2 shows similar results with alternative landscapes and decision rules.

Producers search for entry positions only on the intervals between incumbents (bridge intervals) or on the intervals beyond incumbents (open intervals). Because competition depends only on immediate neighbors, and because the Brownian path is a Markov process, entrants only need to know the positions and appeals of the incumbents on the endpoints of an interval to characterize the optimal entry position on the interval. They select an optimal entry distance Δ from the leftmost competitor (x_l) on bridge intervals (from the nearest competitor on open intervals). For convenience, the distance from their rightmost competitor, at x_r is given by $\bar{\Delta} = x_r - x_l - \Delta$. The expected utility can be decomposed into mean (M) and variance (V) components, taking the form:

$$E(u(\Delta)) = aM(\Delta) - \exp\left(-bM(\Delta) + \frac{1}{2}b^2V(\Delta)\right)$$

$M(\Delta)$ is the expected appeal at Δ, and $V(\Delta)$ is the contribution of variance to the expected utility. On open intervals, these equal:

$$\begin{aligned} M(\Delta) &= A_{X_C} + c(\Delta) \\ V(\Delta) &= \Delta\sigma^2 \end{aligned}$$

On bridge intervals, these equal:

$$\begin{aligned} M(\Delta) &= A_{x_l} + \frac{A_{x_r} - A_{x_l}}{x_r - x_l} \cdot \Delta + c(\Delta) + c(\bar{\Delta}) \\ V(\Delta) &= \frac{\Delta \cdot \bar{\Delta}}{x_r - x_l} \end{aligned}$$

Entrants enter at the interval and position that maximizes their expected utility.

Fig. 3 illustrates this entry process for three entrants. The initial producer (P0) enters at position $x = 0$ with appeal $A = 0$, anchoring the landscape. The full appeal landscape, shown as a gray line, is unobservable to producers. Next, entrant one (P1) evaluates expected appeal over the open intervals left (A) and right (B) of P0 (1 SD of uncertainty in the gray regions), and their expected utility of entering at every point ("Expected Utility"). P1 enters at their position of maximum expected utility, marked by the vertical dashed line, and observes their true appeal, marked by the open circle, which falls below their initial expectation. Finally, entrant two (P2) repeats this process, evaluating their expected appeal on the open interval left of P0 (C), the open interval right of P1 (E), and on the bridge interval between them (D). P2 calculates their expected utility over each interval and enters at the maximum, to the left of P0. Note that P1 and P2 have identical expectations over regions A and C, as P1's entry provided no additional information about them.

[Figure 3 about here.]

3.3 The Categorization Process

While the audience plays no role in defining the appeal of different positions on the landscape, it does exist to categorize clusters of similar producers in the landscape. I model this categorization process by fitting a finite Gaussian mixture model to the positions ($\mathcal{X}_t$) of producers in the market in each period (Dempster et al., 1977; Xu and Wunsch II, 2005). This model assumes that the positions of a set of points, $x_1, \dots x_t$, are given by some set of k normal distributions, $\{N(\mu_1, \sigma_1^2), \dots, N(\mu_k, \sigma_k^2)\}$, centered at different means and with potentially different variances. Each point x has a probability of belonging to each distribution i, $p_i(x)$. Gaussian mixtures place the centers of clusters at particularly dense parts of the set. This model not only partitions the set of producers into categories but also models each producer's grade of membership in each category—positions close to the center of a cluster receive higher grades of membership. The number of clusters k was selected using the Bayesian Information Criterion (Schwarz, 1978; Steele and Raftery, 2009).

[Figure 4 about here.]

Fig. 4 illustrates one simulated market, the clusters identified in it by the Gaussian mixture model, and the associated measures of category membership described further below. Panel **A** shows the positions of producers and the appeals A of their positions; the bottom of the panel shows the local density of producers across the market. Panel **B** shows the probability density functions of the two clusters identified in the market: the two clusters align with the peaks seen in Panel **A**.

Formally, I define the grade of membership GOM of a point x in cluster i as the logarithm of a point's predicted likelihood of being in the cluster, $p_i(x)$, normalized by the peak predicted likelihood of the entire cluster:

$$\mathrm{GOM}_i(x) = -\left(\max_{z\in\mathbb{R}}\left[\log p_i(z)\right] - \log p_i(x)\right)$$

Comparison against the peak-likelihood point controls for differences in cluster width: points in more diffuse clusters have lower likelihoods on average. Panel **C** of fig. 4 shows the grade-of-membership functions associated with each cluster identified in the example: by construction, the most central producers in each clusters have identical grades of membership.

I also construct measures of miscategorization and spanning for each position. For every position, I consider the two clusters in which the point has its highest grades of membership. Taking $p_1(x)$ and $p_2(x)$ to be the highest and second highest predicted likelihoods for the position, I first define a measure of miscategorization and a measure of spanning, as depicted in Panel **D** of fig. 4:

$$\mathrm{Miscat}(x) = 1 - (p_1(x) - p_2(x))$$
$$\mathrm{Span}(x) = \frac{p_2(x)}{p_1(x)}$$

Positions are *miscategorized* either if they are unlikely to belong in any cluster or if they are about equally likely to belong to multiple clusters. Positions are spanners if they are similarly likely to belong in multiple clusters. I reduce these to binary measures, so that a position x is considered miscategorized if $Miscat(x) > 0.5$, and it is considered a spanner if $Span(x) > 0.01$. The particular thresholds were chosen to ensure a balance of producers across classes, but analysis is robust to variation around these specific values. Finally, I construct a binary measure of *non-categorization* if a position is miscategorized and is not a spanner. This allows me to examine the effects of spanning and non-categorization separately.[7]

[7] The regressions I consider below include *Spanner* and *Non-Categorized* as predictors. Using *Spanner* and *Miscategorized* as predictors instead causes the coefficient on spanner to reflect the effect of spanning net of the underlying effect of miscategorization. All else equal, spanning points tend to occur in denser, higher value regions in which two clusters exist close together, which causes *Spanner* to predict a positive effect of spanning relative to the miscategorization alone. Treating spanning and non-categorization as mutually exclusive allows for closer comparison to prior research.

4 Results

4.1 Reproduction of Miscategorization Effects

I reproduce the core claims of category theory at three levels. First, at the most fundamental level, the miscategorization penalty argues that producers with a higher grade of membership in a category appeal more to members of the audience. Positions that do not fit well into any category should underperform those that do. Second, as a direct replication of category theory, I test whether category spanners are less appealing than single-category specialists. Finally, I examine the direct mechanism by which this effects appears in the landscape model: success generates herding, which generates categorization; conversely failure isolates producers. I measure whether more isolated producers—those with a higher minimum distance to their nearest neighbor—tend to underperform less isolated producers.

I test these relationships using cross-sectional linear regressions estimating the effect of each measure of category membership on the appeal of the position. I estimate the effects within each period of each market. Specifically, within each period t and each market i, I estimate three regressions:

$$
\begin{aligned}
A_{x,i,t} &= \alpha + \beta_{i,t} NGOM_{x,i,t} + \epsilon_{x,i,t} \\
A_{x,i,t} &= \alpha + \beta_{1,i,t} NoCategory_{x,i,t} + \beta_{2,i,t} Spanner_{x,i,t} + \epsilon_{x,i,t} \\
A_{x,i,t} &= \alpha + \beta_{i,t} Distance_{x,i,t} + \epsilon_{x,i,t}
\end{aligned}
$$

Here, x indexes all producers within a market during the period, and each regression considers one of the measures of category membership: (negative) grade of membership; binary miscategorization; or distance to the nearest neighbor. Thus, in each market and each period, these regressions produce three separate β coefficients representing how miscategorization predicts producer appeal.

In each case, negative β indicates a miscategorization penalty. Fig. 5 plot these coefficients across all markets over time. Fig. 5a shows the difference between category-spanning and non-category-spanning positions; fig. 5b shows the effect of decreasing grade-of-membership; Fig. 5c shows the effect of an increasing distance to a neighbor on producer appeal. The plots show the average effect (bold), the 10th and 90th percentiles (dashed), and all individual markets (thin gray).

[Figure 5 about here.]

Each of the figures supports the predicted relationships. Category spanners generally have lower appeal—the estimated effect of spanning is generally negative within markets and it is negative when aggregating across all markets. Positions with low grade of membership in every category have lower appeal. Most fundamentally, isolated positions have lower appeal. These relationships appear to strengthen over time as markets exhaust obvious opportunities in the landscape.

Miscategorization penalties, however, regularly disappear. As the thin lines of fig. 5 suggest, many individual markets experience periods when the correlation between clustering and appeal inverts. In markets at least 50 periods away from initial conditions, about 6 percent of markets will experience an inversion of the penalty in grade of membership; 49 percent of markets experience such an inversion at some point in their history. With the penalty in category spanner status, 14 percent of markets experience an inversion in any given period, and 70 percent experience it in their history. Inversions occur when the most successful producers in a market are located outside of major clusters or categories. They occur entirely because the landscape is unknown: as producers discover highly appealing regions outside of existing clusters, they find themselves simultaneously outside of existing categorizations and in positions of high appeal. Once enough producers flock to these outside positions, the overall predictive power of categories disappears.

4.2 Change and Emergence of Categories over Time

The Gaussian model of categorization adapts each period as producers enter new positions in the landscape. These adaptations take two forms: first, new categories can emerge or disappear as the model fit favors a larger or smaller number of mixture distributions; second, the distributions themselves shift as producers uncover more of the landscape, affecting the grades of membership of existing producers. Both adaptations are predictable.

4.2.1 Category emergence and disappearance

At its core, the miscategorization penalty in this model arises because producers position themselves in regions of high appeal, so that producer density is predictive of local appeal—in effect, producer clusters reflect a lay theory of the market landscape. New entrants into the landscape either confirm or refute this theory. They confirm it by succeeding within existing clusters or by failing outside those clusters. They refute the theory by finding success outside existing clusters. Refutations weaken the lay theory and the miscategorization penalty; at the same time, they uncover regions of the landscape that may form the core of future categories. Refutations can also reveal that neglected valleys between peaks of the landscape are more appealing than initially presumed, causing producers to fill the gaps between existing clusters, merging two categories together.

If this argument holds, then weakening of the miscategorization penalty (i.e. negative trends in the penalty) will predict changes in the number of categories. I define a penalty trend as the change in the penalty β over the prior 25 periods, evaluating the penalty as described in the prior section. A negative trend implies that the miscategorization penalty has weakened; a positive trend, that the penalty grew stronger. I then predict the probability that the number of categories will change in the current period as a function of recent trend.

[Table 1 about here.]

Tbl. 1 presents the estimates of these logistic regressions. The table shows that a recent positive trend (strengthening penalty) strongly predicts stability in the number of categories in a given period. A weakening penalty thus predicts a change—an increase or decrease—in the number of categories.

Insofar as the miscategorization penalty and its trends are directly observable in markets, these results helps contextualize various empirical findings in the category theory literature. Markets with a weaker or weakening miscategorization penalty indicate the presence of untapped opportunities, with potential for further exploitation. In particular this result presents an alternate interpretation of the market-maker/market-taker distinction drawn by Pontikes (2012). Pontikes argues that different audiences may have different tastes for categorical ambiguity, showing that venture capitalists are more favorable to category spanners than are general audiences. The landscape model suggests that venture capitalists may select for systematically different markets than the general audience. If venture capitalists elect to operate in markets with a weaker penalty, their preference for categorical ambiguity reflects the decreased predictive power of categorical labels in such markets—in such markets the penalty itself is weaker. Similar dynamics may cause the preference for atypical hedge funds identified by Smith (2011).

4.2.2 Category movement

Producers maintain a shared expectation of the value of all positions in the landscape. They enter at the position that maximizes their expectation. What they discover when they enter constitutes not only a shock to their own expectation about the region they entered, but also to the expectations of all subsequent producers. Positive shocks attract subsequent entry, and negative shocks deter entry. Because categories move towards positions of increased producer density, entry shocks cause categories to shift and affect the categorical membership of incumbent producers.

The landscape model provides direct insight into producer expectations. I construct a z-scored measure of the difference between realized and expected appeal:

$$z_x = \frac{A_x - E(A_x)}{\sqrt{V(x)}}$$

Here, V is the variance of the Brownian walk at x conditional on the positions of its nearest neighbors.

I focus on the entrant themselves and track how the shock z_x affects their grade of membership and likelihood of miscategorization in subsequent periods.

[Figure 6 about here.]

Fig. 6 plots the effects of entry surprise on categorical centrality over time, highlighting the effect of positive ($z_x = 1, 2$) and negative surprises ($z_x = -1, -2$). The plot confirms that positive shocks increase the categorical centrality of the entrant over time, while negative shocks reduce it. There is a notable asymmetry between positive and negative shocks: Because entrants are already entering at the position of highest expected utility across the landscape, positive shocks have only a limited ability to draw in subsequent entrants. Negative shocks, on the other hand, do serve to dissuade further entry, especially if they push the expected appeal of the nearby region below the expected appeal of the next-best alternative.

As with category emergence, this result directly suggests alternative explanations for prior findings in the category literature. In particular, Zhao et al. (2018) show how hit video games establish proto-categories that develop into established categories as they attract imitators. The landscape model describes how unexpected hits provide the seeds for novel categories, as well as the pace at which market audiences come to recognize such developments.

4.3 Competitive Regimes in the Landscape Model

In the landscape model described here, reproduction of the miscategorization penalty relies on the interaction of two mechanisms: (1) the tendency of producers to cluster in regions of high appeal; and (2) the tendency of audiences to categorize dense clusters of producers. The first mechanism depends on the specification of competition in the market. As described above, the model assumes that all producers exert equivalent competitive pressure on each other. If instead high-appeal producers exert greater competitive pressure than low-appeal producers, they may deter and exclude potential competitors, restricting new entrants to a low-appeal periphery (cf. resource-partitioning, Carroll, 1985). In this case, the audience would assign categories to low-appeal positions, reversing the miscategorization penalty.

I model this variant by making the competition function exponential in producer appeal rather than constant, $c(x, \mathcal{X}_t) = \frac{\exp A_{x_l}}{x - x_l} + \frac{\exp A_{x_r}}{x_r - x}$. Fig. 7a plots one such simulated market, showing how entrants avoid high-appeal competitors. Fig. 7b shows that the miscategorization penalty (in grade of membership) reverses in such situations, becoming a miscategorization bonus instead.

[Figure 7 about here.]

This result primarily suggests that the miscategorization penalty is particularly sensitive to specific interactions between categorization processes and competitive structure. In markets that resemble the situation described here, it should be possible to empirically observe an analogous reversal of the penalty. If instead audience categorizations follow categorical exemplars rather than similarity-based prototypes, the miscategorization penalty may persist (Murphy, 2004; Foster-Hanson and Rhodes, 2019).

5 Discussion & Conclusions

This paper proposes a landscape model as a null model for the appearance of miscategorization effects in markets. Category theory has predominantly relied on a mechanism in which market audiences, applying categorical rules for market membership, punish or ignore deviant producers and create an empirically observable penalty against category-spanning and miscategorized products. This account struggles not only to explain how cognitively flexible individuals (Durand and Paolella, 2013; Guilbeault et al., 2021) come together into a rigid market collective but also to account for how collective categories shift over time. The landscape model accounts both for the cross-sectional appearance of collective penalties and for the dynamics of categories in markets: As producers compete in an uncertain, rugged landscape, they uncover and herd around positions of high appeal. As audiences label clusters of similar producers, they will find that these labels correlate with appeal, while low-appeal producers find themselves isolated and unclassified. As producers explore the environment, the categories themselves will emerge, disappear, and shift in predictable ways.

The landscape model directly implies a number of testable predictions that may serve to substantiate its assumptions. Cross-sectional analyses replicating the miscategorization penalty predict: (1) Markets should experience a regular inversion of the miscategorization penalty, which declines with market maturity (c.f. Paolella and Durand, 2016). (2) Miscategorized producers should be more isolated or distinct in product space than well-categorized producers. Dynamic analyses predict: (3) A weakening miscategorization penalty should predict categorical emergence. (4) Entry shocks shift entrants' categorical centrality over time, and negative shocks reduce centrality more than positive shocks raise it.

As a null model, the landscape model also helps characterize the specific mechanism by which cognitive categorization effects operate. The model assumes that categories preferentially attach to clusters of dense, similar producers—an assumption shared by category theory (Hannan et al., 2019). This leads directly to the prediction that miscategorization penalties should systematically reverse in resource partitioned markets. Market category systems may operate differently: a consistent finding in markets where peak producers are able to successfully exclude competitors (e.g. resource partitioning, patent protection) may suggest that audiences rely on exemplar-driven categorizations (Murphy, 2004). Further still, category systems that label features orthogonal to producer success (e.g. exhaustive, mutually exclusive systems of neutral product features) should be immune to miscategorization effects under the landscape model.

The landscape model also helps direct the attention of category research onto the search behavior of producers. In this model, categories lag the search efforts of producers, imposing excessive miscategorization on distinct producers relative to the underlying fitness of their positions. First, in the presence of miscategorization effects, producers should prefer inefficient categorically-favored strategies to efficient categorically-disfavored strategies. This preference can measure the strength of any categorical hysteresis, and may be directly testable through cross-national comparisons of analogous markets. Second, insofar as the landscape model suggests that categories serve to facilitate terse communication, compression of the communication channel from producers to audiences should magnify the strength of cognitive categorical effects relative to landscape effects insofar as audiences become more reliant on categorical imputations (Leung and Sharkey, 2013, suggests that such effects exist). Producer behavior should be more categorically-driven in time-constrained or transient markets.

The model presented here is by no means the final word on landscape models of categorization. This model presents a narrow view of categorization processes that does not accommodate for category nesting (Cudennec and Durand, 2022), or fully model the process of categorical continuity and disappearance. More importantly, the model examines only the producer entry process, and does not consider the possibility that producers can exit the market or embrace learning strategies within it in order to enable cheaper search (cf. Ries, 2011). Neither does the model measure market participants' differential ability to remember the outcomes of past forays into the market. It also ignores the multidimensionality of product spaces, which may be expected to increase the relative number and appeal of distinct positions (Péli and Nooteboom, 1999; Weinberger, 1991). Each modification offers an avenue for further elaboration of category dynamics within markets. The model also makes specific assumptions about producer risk aversion and their search behavior in novel regions of the marketplace. While the exact form of these assumptions is not critical to the results of the paper, further research on producer search behavior can contribute both to category theory and to the literature on organizational exploration.

Finally, the model takes its place alongside the production-side theory of miscategorization effects described by Hsu (2006) and Hsu et al. (2009). There the principle of allocation argument presupposes the existence of distinct categories representing audiences with separable tastes. Miscategorization penalties arise from producers' inability to appeal to multiple distinct categories, and diversification imposes an efficiency penalty relative to specialization. The distinction between the principle of allocation argument and the rugged landscape model may be understood through their underlying assumptions about market topology. The principle of allocation assumes that markets can be partitioned into distinct segments, whereas the landscape model interprets segment labels as partial views of a latent feature space. Intermediate positions in the principle of allocation argument necessarily involve blends across segments, whereas intermediate positions in the landscape model represent efforts to identify intermediate audiences. In the landscape model, miscategorization penalties arise from the probabilistic inferiority of such intermediate positions.

In summary, this paper presents a null model relying on rugged landscapes to account for oft-identified miscategorization effects in markets. This paper emphasizes that producer search behavior in uncertain competitive environments will drive a pattern of herding that provides anchors for an audience categorization process. Such anchoring suffices to produce the appearance of a miscategorization penalty without audience categorizations imposing any additional harm. Coevolution of producers' mental maps of the market and audience categorizations creates testable predictions about categorical dynamics. This model does not exclude the possibility of cognitive effects, and it invites research into how audience cognition manifests through its effect on producer search.

6 References

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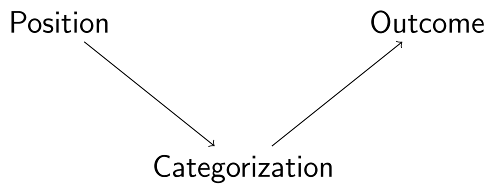
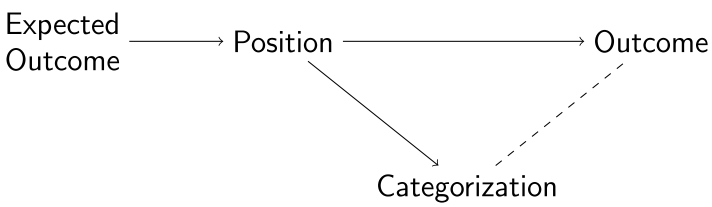
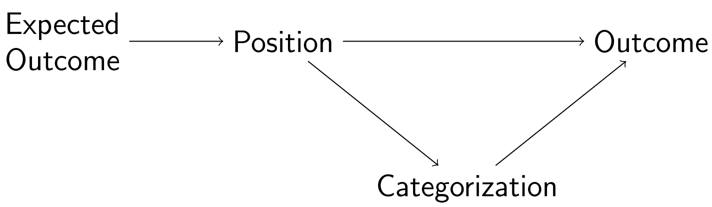


(a) Predominant cognitive categorization model: Producer positioning affects market outcomes through categorization schemas



(b) Rugged landscape null model: Producers position in expectation of outcomes. Positioning reveals landscape outcomes and induces categorization, creating a spurious correlation.



(c) Integrated model: Categorization partially mediates outcomes beyond landscape effects

Figure 1: Differing causal logics of possible category theories.

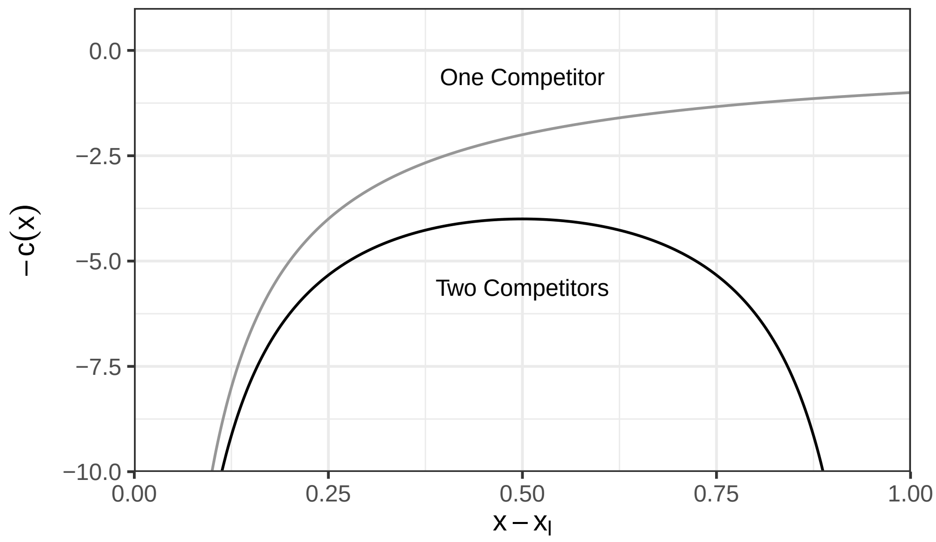


Figure 2: Competitive penalties, with single competitor at $x = 0$ or multiple at $x = 0, 1$

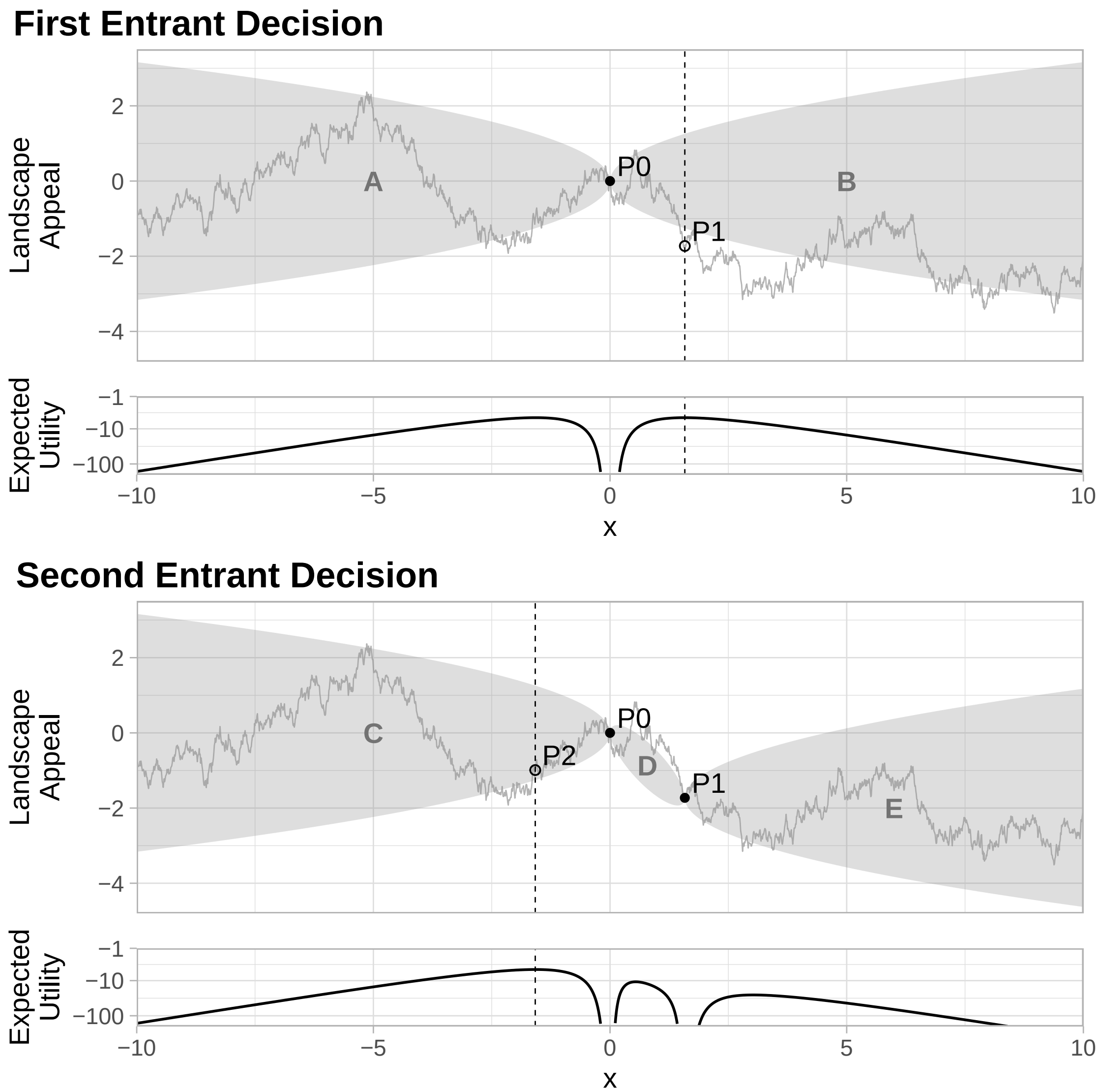


Figure 3: Illustration of the entry process for three entrants.

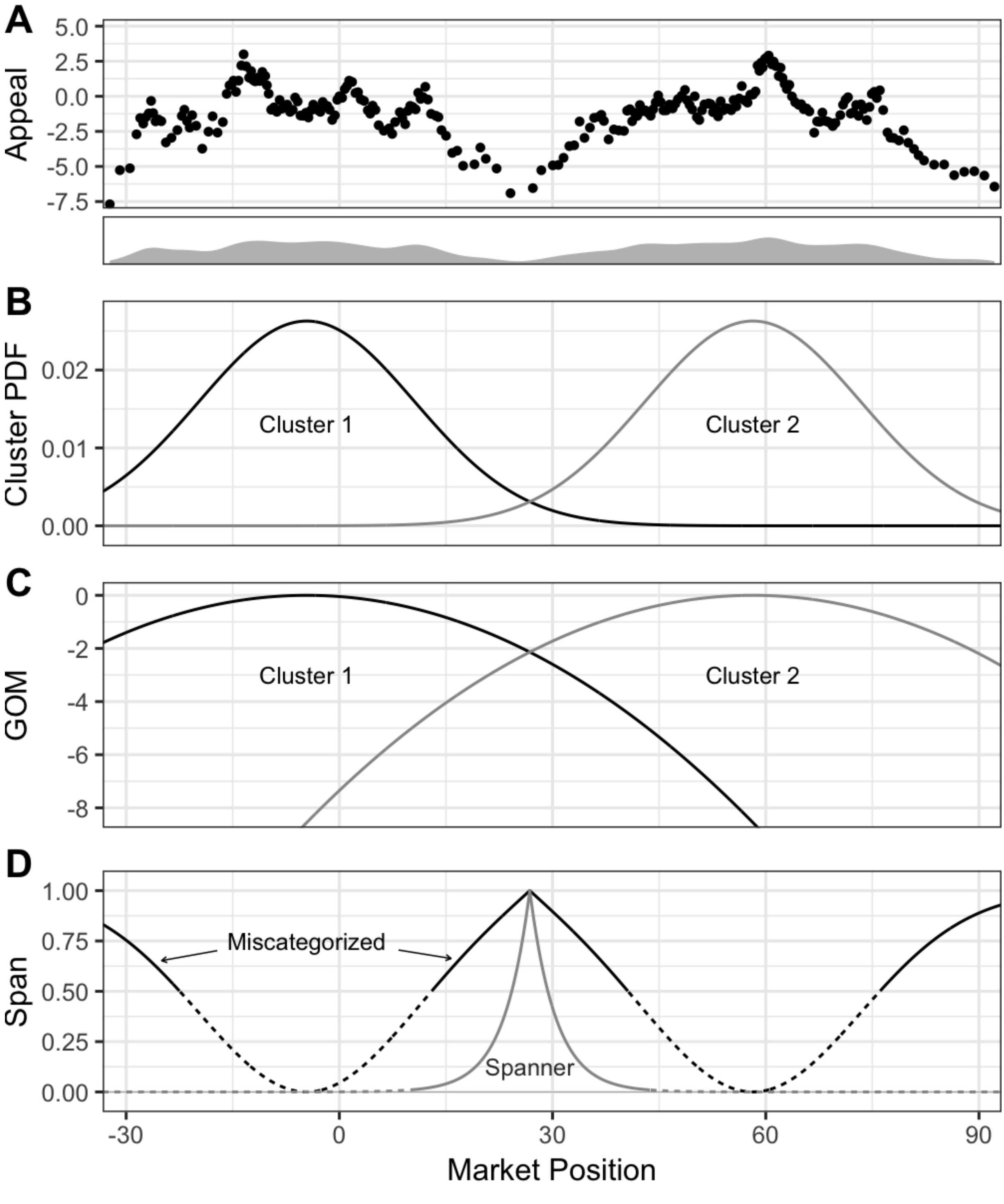
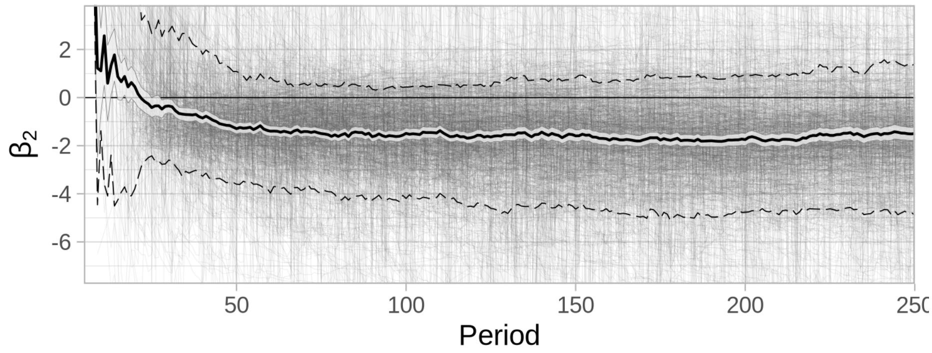
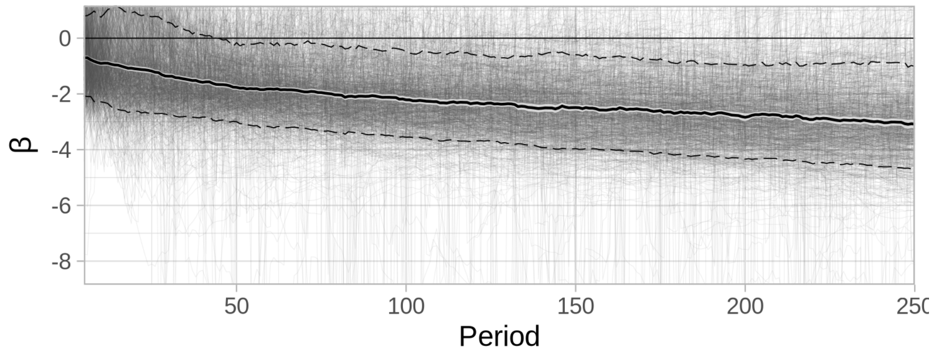


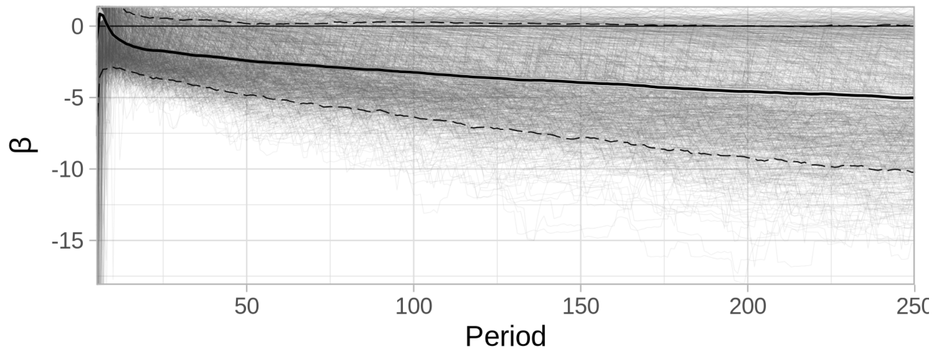
Figure 4: Example of a simulated market, estimated clusters, and measures of category membership



(a) Effect of category spanning. $W = \alpha + \beta_1 \cdot NoCategory + \beta_2 \cdot Spanner$

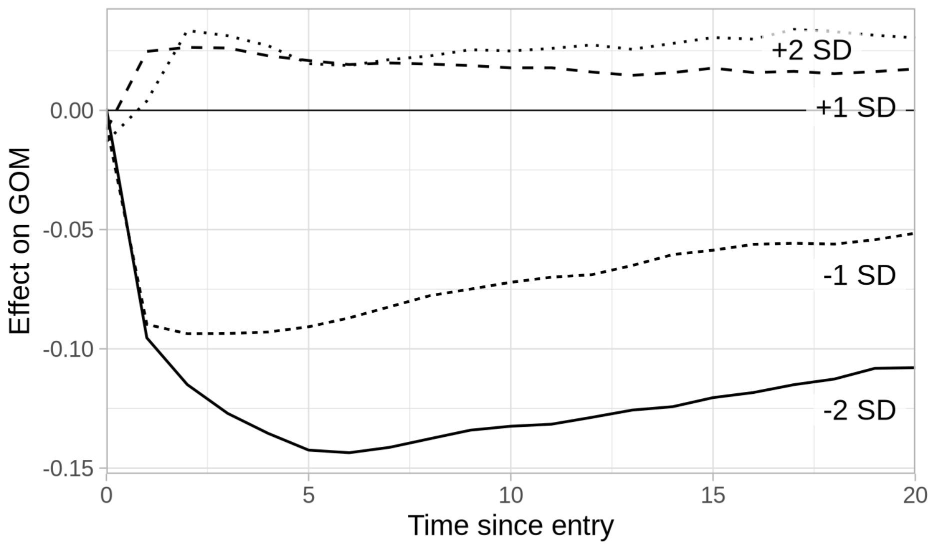


(b) Effect of decreasing grade of membership. $W = \alpha + \beta \cdot NGOM$

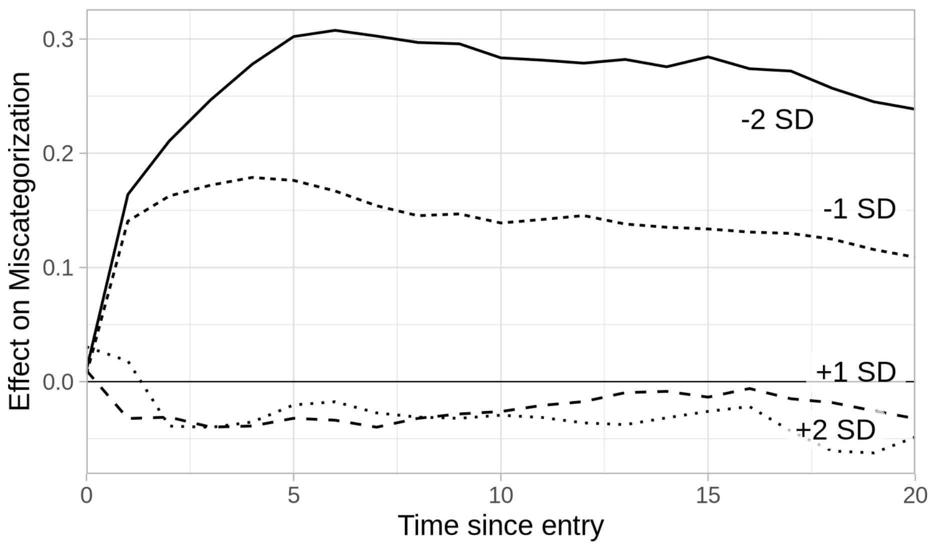


(c) Effect of increasing distance to neighbor. $W = \alpha + \beta \cdot distance$

Figure 5: Effect of position characteristics on position appeal, 1000 simulated markets, 250 periods. Average effect in bold, 95% confidence interval in white band. Dashed lines indicate 10th and 90th percentiles of effect across all markets.

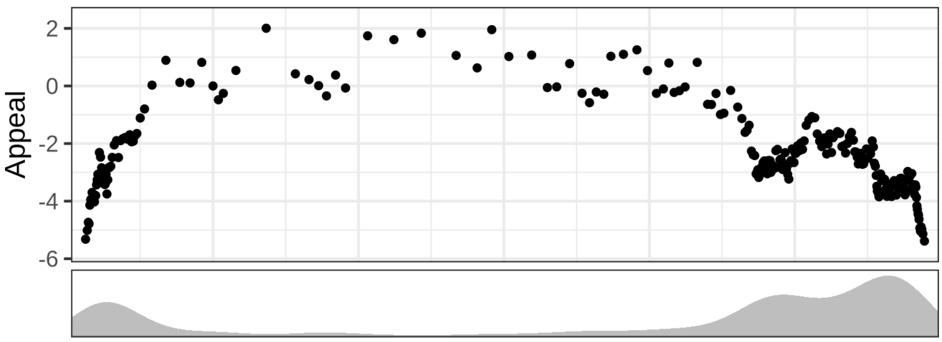


(a) Effect of entry surprise on grade of membership

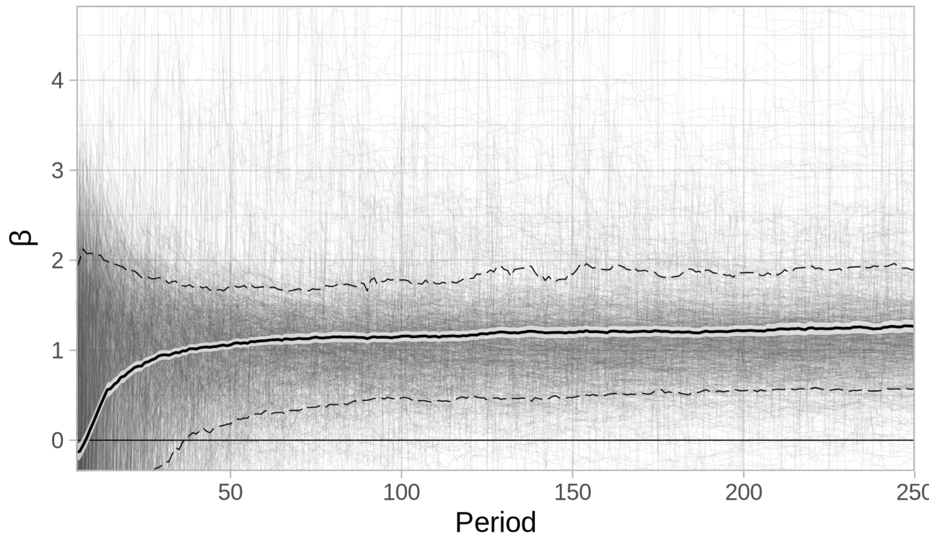


(b) Effect of entry surprise on likelihood of miscategorization

Figure 6: Effect of entry appeal surprise on categorical centrality.



(a) Producer appeal and positioning in a simulated market.



(b) Effect of decreasing grade of membership on appeal. $W = \alpha + \beta \cdot NGOM$

Figure 7: Category effects in partitioned markets.

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	-3.013^{***}	-3.015^{***}	-3.204^{***}	-3.202^{***}	-3.204^{***}
	(0.010)	(0.010)	(0.014)	(0.014)	(0.014)
Penalty Trend (Distance)	-0.637^{***}				
	(0.088)				
Penalty Trend (GOM)		-1.207^{***}			
		(0.116)			
Penalty Trend (No Cat.)			-0.629^{***}		-0.655^{***}
			(0.071)		(0.074)
Penalty Trend (Spanner)				-0.052	0.130
				(0.090)	(0.092)
Num. obs.	221000	221000	138193	138193	138193

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 1: Trends in the clustering penalty predict category change

Appendix 1: Simulation Details

This appendix discusses the details of the simulation and the derivation of its key equations. Minimal representative code is available in a repository at (https://anonymous.4open.science/r/os-categories-42D7).

The simulation models the course of 1000 markets over 250 periods. In each market, producers can take positions along the real number line. The key characteristic of each market is the appeal function assigning producer appeal to specific positions along the number line. This appeal function is given by a Brownian walk A with drift parameter $\mu = 0$ and variance parameter $\sigma^2 = 1$. Zero-drift ensures that there's no overall direction of improvement to the market, while the choice of variance is arbitrary. Each simulated market is independent, with its specific course determined by its realization of the Brownian walk.

Because Brownian walks exhibit infinitesimal variation, the landscape is not pre-generated but is sampled as producers enter it. The sampling rule is derived from the basic properties of Brownian motion (Revuz and Yor, 1999). Brownian walks are Markovian and Gaussian, so a sample at a focal position is drawn from a normal distribution depending on the fitness values of the known positions to the immediate left and right of the focal point—when the focal point has neighbors on only one side (i.e. is the leftmost or rightmost point), its distribution depends only on the fitness of the sole nearest neighbor. I describe the specific distribution function below.

I seed each market with an initial producer at $x = 0$ with appeal $A = 0$. In each subsequent period, one producer enters the market at some position. This producer can observe the positions of all previous entrants as well as the value of A at those positions. The new entrant chooses her entry position by conditioning on this prior knowledge; once she enters, she discovers the value of A at her chosen position, both for herself and for any subsequent entrants. An entrant enters at the position that maximizes her expected utility, as described below.

An entrant can enter either between two existing positions or at the extremes of the market (i.e. left of the leftmost producer or right of the rightmost producer). If the entrant enters at the edge of the market, we can denote the closest producer by x_C, with fitness given by $A(x_C)$, and denote the entrant's chosen position by δ if the entrant chooses to enter at $x_C + \delta$ (or at $x_C - \delta$ on the left of the market). If the producer enters between two existing positions, we can denote the leftmost and rightmost neighbors by x_l and x_r, with their fitnesses given by $A(x_l)$ and $A(x_r)$, and we can denote the entrant's chosen position by δ where the entrant chooses to enter at $x_l + \delta$. In period t of a market, t producers have entered the market, so that the new entrant must consider entry in $t + 1$ different intervals: 2 intervals at the left and right of the market (open intervals), and the $t - 1$ intervals between each existing pair of producers in the market (bridge intervals). In each of these $t + 1$ intervals, we can thus consider an optimal entry position denoted by δ.

In the Brownian walk fitness landscape, the value of a given position follows a normal distribution with mean and variance given by distance from and values of known positions. On an open interval, the fitness of entry at $x_C + \delta$ follows an unconstrained Brownian walk:

$$A(x_C + \delta) \sim N\left(A(x_C), \delta\right)$$

On a bridge interval (between x_l and x_r), the fitness of entry at $x_l + \delta$ follows a Brownian bridge:

$$A(x_l + \delta) \sim N\left(A(x_l) + \delta\frac{A(x_r) + A(x_l)}{x_r - x_l}, \frac{\delta(x_r - x_l - \delta)}{x_r - x_l}\right)$$

Each producer derives utility from the amount of income y they receive at a given position according to the utility function $u(y)$. Within each interval, we define the mean function $M(\delta)$ and variance function $V(\delta)$. The expected utility at δ is given by:

$$U(\delta) = E\left[u(M(\delta) + \sqrt{V(\delta)}Z)\right]$$

The expectation is taken over Z, a standard normal variable. Chipman (1973) describes conditions on the utility function u that ensure that the expected utility exists. The utility functions I use here (and describe below) satisfy the conditions.

The entrant picks an optimal position within each interval by picking δ to maximize expected utility. For twice differentiable u, and differentiable M, V, these positions can be identified by setting the first derivative to zero, giving the following criterion condition:

$$
\begin{aligned}
0 = U'(\delta) &= E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]M'(\delta)+ \\
&\quad + \frac{1}{2}\frac{V'(\delta)}{\sqrt{V(\delta)}}E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\cdot Z\right] \\
&= E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]M'(\delta)+ \\
&\quad + \frac{1}{2}V'(\delta)E\left[u''\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right] \\
&= E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]\cdot \\
&\quad \cdot\left[M'(\delta)+\frac{1}{2}V'(\delta)\frac{E\left[u''\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]}{E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]}\right]
\end{aligned}
$$

If $u' > 0$ everywhere, i.e. marginal utility is declining in income, the relevant criterion reduces to the second component:

$$
0 = M'(\delta) + \frac{1}{2}V'(\delta)\frac{E\left[u''\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]}{E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right]} \tag{1}
$$

With a specific utility function u and precise specifications for M and V, (1) can be simplified further.

Here, the actor's utility in income is given by

$$
u(y) = ay - \exp(-by)
$$

With this u, the expected utility function U and its derivatives reduce to:

$$
\begin{aligned}
E\left[u\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right] &= u\left(M(\delta)-\frac{1}{2}bV(\delta)\right)+\frac{1}{2}abV(\delta)\\
&= aM(\delta)-\exp\left(-bM(\delta)+\frac{1}{2}b^2V(\delta)\right)\\
E\left[u'\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right] &= u'\left(M(\delta)-\frac{1}{2}bV(\delta)\right)\\
&= a+b\exp\left(-bM(\delta)+\frac{1}{2}b^2V(\delta)\right)\\
E\left[u''\left(M(\delta)+\sqrt{V(\delta)}Z\right)\right] &= u''\left(M(\delta)-\frac{1}{2}bV(\delta)\right)\\
&= -b^2\exp\left(-bM(\delta)+\frac{1}{2}b^2V(\delta)\right)
\end{aligned}
$$

The mean and variance functions M and V derive from the characteristics of the Brownian walk W and a competition function c indicating the loss of income due to competition from nearby producers. Here I use

$$
\begin{aligned}
c(\delta) &= -\frac{1}{\delta}\\
c'(\delta) &= \frac{1}{\delta^2}
\end{aligned}
$$

On an open interval, in which the entrant has only one immediate neighbor at x_C, M and V take simple forms:

$$
\begin{aligned}
M(\delta) &= E\left[A(x_C+\delta)\right] + c(\delta) \\
&= A(x_C) + c(\delta) \\
M'(\delta) &= c'(\delta) \\
V(\delta) &= \delta\sigma^2 \\
V'(\delta) &= \sigma^2
\end{aligned}
$$

On a bridge interval, the entrant has a left neighbor at x_l and a right neighbor at x_r. With δ the distance from x_l, we denote the distance from x_r by $\bar{\delta} = x_r - x_l - \delta$. M and V take the following forms:

$$
\begin{aligned}
M(\delta) &= E\left[A(x_l+\delta)\right] + c(\delta) + c(\bar{\delta}) \\
&= A(x_l) + \frac{A(x_r) - A(x_l)}{x_r - x_l}\delta + c(\delta) + c(\bar{\delta}) \\
M'(\delta) &= \frac{A(x_r) - A(x_l)}{x_r - x_l} + c'(\delta) - c'(\bar{\delta}) \\
V(\delta) &= \frac{\delta\bar{\delta}}{x_r - x_l}\sigma^2 \\
V'(\delta) &= \frac{\bar{\delta} - \delta}{x_r - x_l}\sigma^2
\end{aligned}
$$

Substituting these values into (1) allows us to solve for the entrant's optimal entry point on each possible interval. The ratio of expected utilities $\frac{Eu''}{Eu'}$ is always negative here, and in practice the location of this root reflects the declining marginal effect of competition (M') matching the growing marginal effect of variance (V').

Finally, I identify these optima numerically with R's *uniroot* command (R Core Team, 2016). The presence of the exponential function in u tends to cause numerical instability for extremely large or extremely small δ. In practice, (1) can be easier to solve after logarithmic transformation, looking for roots of the criterion

$$
\begin{aligned}
0 &= -\log M'(\delta) + \log\left(-\frac{1}{2}V'(\delta)\frac{Eu''}{Eu'}\right) \\
&= \log V'(\delta) - \log(2) + \log\left(-\frac{Eu''}{Eu'}\right) - \log M'(\delta)
\end{aligned}
$$

with care taken to ensure that optimization is done over positive values of M' and V'.

Once the entrant has identified optimal entry points on each subinterval, she enters at the point with the highest overall expected utility.

References

Chipman, J. S. (1973). The Ordering of Portfolios in Terms of Mean and Variance. *Review of Economic Studies*, 40(2):167–190.

R Core Team (2016). R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing.

Revuz, D. and Yor, M. (1999). *Continuous Martingales and Brownian Motion*. Springer-Verlag, Berlin, 3rd edition edition.

Appendix 2: Alternative Assumptions for the Landscape Model

This appendix discusses various alternative constructions for a landscape model and the consequences of alternative assumptions.

Ruggedness and Uncertainty

A major question with the landscape model of category formation is whether uncertainty and ruggedness are necessary to the results. A substantial economic literature identifies agglomeration and clustering effects even in the absence of exogenous variation of the environment, suggesting that some model of producer clustering could be constructed in the absence of exogenous environmental variation. Canoy and Peitz (2003), for instance, model a market in which equivalent producers differentiate both horizontally and vertically in quality, given a particular order of entry and perfect rationality. While this would certainly be an interesting addition to the category literature, I argue that the assumption of environmental variation imposes fewer constraints on the applicability of the theory to real world settings. In effect, this article asks the question of how much of category theory should emerge automatically in a variable, complex environment.

A deeper question concerns the extent of insight producers have into the environment—how uncertain are they about the potential value of positions? In effect, a strong version of cognitive category theory can be understood as assuming that producers operate in such a competitive landscape, barring the effect of audience categorizations on their outcomes. In such a situation, sequential entry by producers will serve to fill out the landscape in order from positions of the highest to the lowest appeal, with adjustment for competitive pressure. In effect, though, each new entrant will either serve to increase the density of existing high appeal positions, or to move further into the valleys of the terrain. In this case, each new entrant will increase the predictive power of the categorization system: while the static predictions of category theory will be replicated, there will be none of the dynamic effects described in the article. Fig. 1 illustrates the evolution of the miscategorization penalty in a deterministic market featuring a single peak whose shape is known to all producers.

In effect, such a landscape serves as the foil against which a rugged landscape theory of market categories can be compared. In the absence of strategic action by market actors to change the

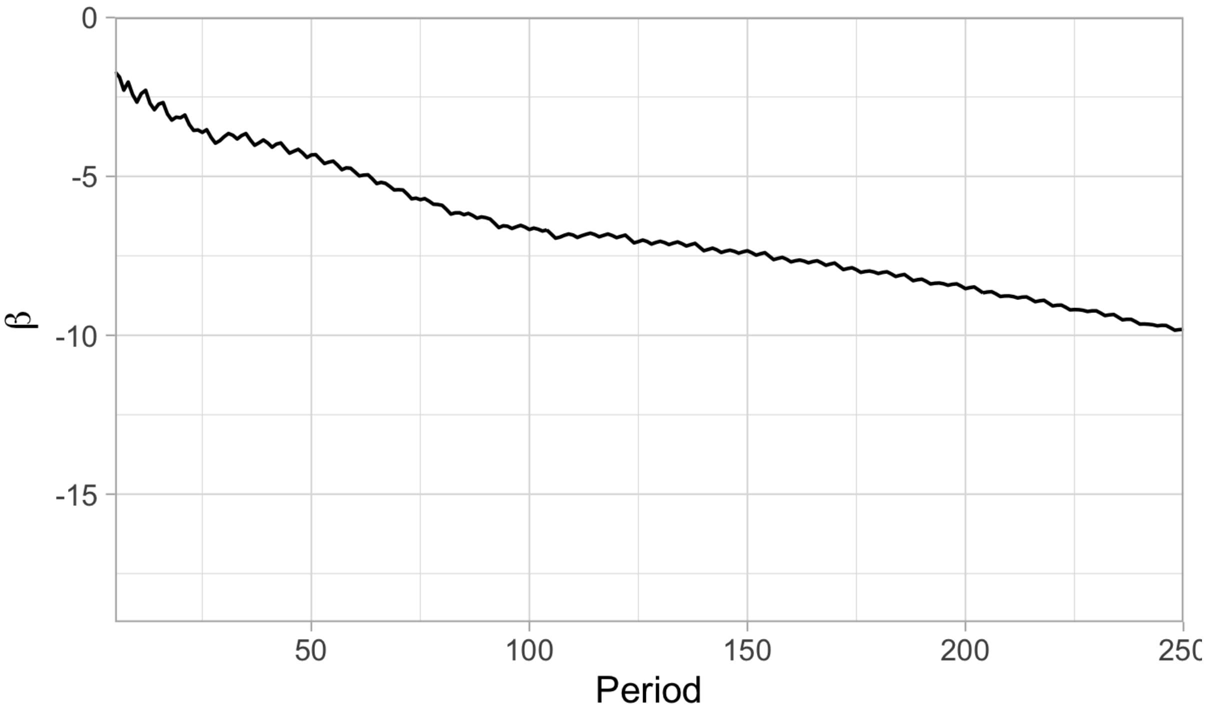


Figure 1: Effect of decreasing grade of membership, deterministic model. $W = \alpha + \beta \cdot NGOM$

nature of categories in the marketplace, producers' awareness of categorical structures should allow them to position in a manner consistent with the way they would exploit a peaked, uncertainty-free landscape.

Salop Model

A key observation of work on competitive positioning in work going back to Salop (1979) is that results of positioning models can be critically driven by competitive behavior at the endpoints of the landscape. In the case of Hotelling (1929), the market was truncated at the endpoints of an interval; in the case of the model in the article, an infinite landscape generates a number of modeling obstacles through the possibility of infinite differentiation. Salop (1979) instead proposed a circular landscape that avoids the endpoint problem by construction. It is likewise possible to simulate the model in this paper on a circular model rather than on an infinite landscape. This imposes a requirement to specify the size of the landscape, which ends up constraining the number of possible categories within the market as well as the period of time in which category dynamics can play out. It remains to the reader to determine whether these concerns outweigh concerns involved in modeling an infinite landscape.

I simulate a version of the model in a Salop market. The model is otherwise identical to that described in the main text: in effect, the entire landscape consists of a Brownian bridge pinned to equal values of appeal at the endpoints. The market is 200 units wide—this is approximately equal to the median market span achieved in the primary model, so that some markets end up more constrained than they otherwise would be, and some less. Fig. 2 shows the miscategorization penalty as measured by grade of membership. In this and other ways, the circular model produces qualitatively similar results to the main model.

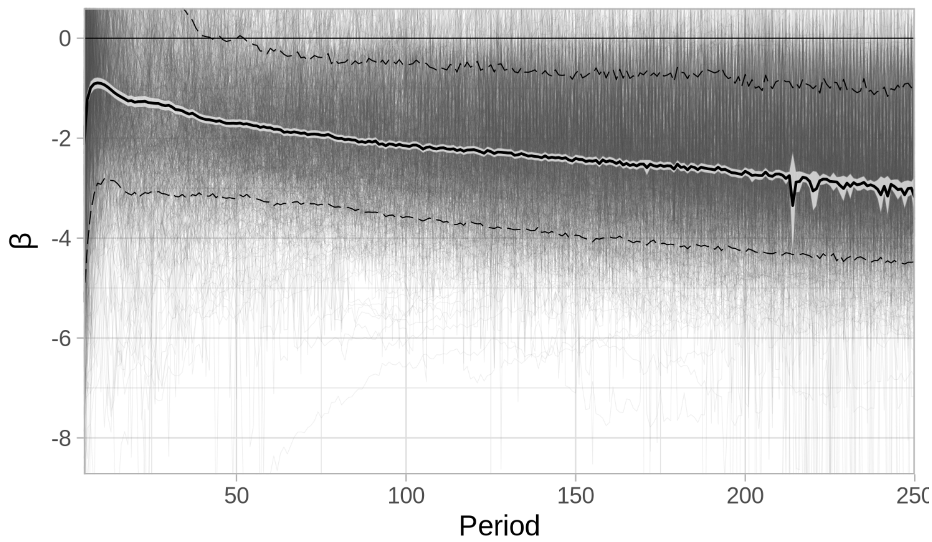


Figure 2: Effect of decreasing grade of membership, Salop model. $W = \alpha + \beta \cdot NGOM$

Boundedly Rational Model

One concern with the landscape model described in the body of the article is that the assumption of producer rationality is excessively strong, especially in light of the long-standing tradition in organizational theory to acknowledge limits to rationality (Simon, 1947). In this particular case, I contend that the rationality assumption serves as a simplifying rather than a constraining assumption. Insofar as the rationality assumption does not impose unreasonable foresight on producers in the model, it instead serves to abstract away from specific assumptions about producer decision-making, and the issues this might raise in analyzing the any set of results.

Nevertheless, it is certainly possible to eliminate the rationality assumption. The key process at work in this model is that entrants seek to imitate successful incumbents. Mechanically, the biggest obstacle is the question of optimal entry position on an open interval. As discussed in the article, competitive pressure on open intervals encourages producers to take infinitely distant positions to differentiate themselves from competitors: there is no optimum at finite distance. Under the rationality assumption, this is resolved by imposing risk aversion on producers. Under a boundedly rational model, it is possible to forego even a utility function and instead impose some heuristic for entering open intervals.

I model one particular variant of a boundedly rational entry process: producers consider an entry position on each possible interval between existing competitors. On open intervals, they consider a position one unit away from the existing competitor. On bridge intervals, they consider the position midway between competitors. In all cases, producers enter at the entry candidate with the highest expected value according to the Brownian walk: in this case, even the variance of the Brownian walk plays no role in the model. Once again, I simulate 1000 markets over 250 periods. Fig. 3 plots the miscategorization penalty as measured by grade of membership, replicating the effect in the main article.

In general, insofar as a boundedly rational entry rule replicates the key mechanism of imitative clustering and achieves reasonable behavior on the extremes of the market, it is likely to replicate the results of the paper. Once again, though, it is unclear whether the rationality assumption imposes excessive theoretical burden relative to the needs of justifying a particular boundedly rational decision process.

Multinomial Logit

A major modeling choice within the paper is the decision to represent competition as a highly simplified force pushing new entrants away from incumbent producers. Though this approach is analytically simple, it lacks a sound competitive microfoundation. This appendix presents an alternative grounding for the competitive market in which producers operate, with a stronger foundation in the economic literature on product variety.

Consider a market represented by the real number line. A finite number (possibly zero) of producers are located at discrete points along the line, x_i, $i \in \{1, \ldots, n\}$, ordered so that $x_j \geq x_i$ if $j > i$. Each producer has a quality A_i, given by a Brownian walk, anchored at some

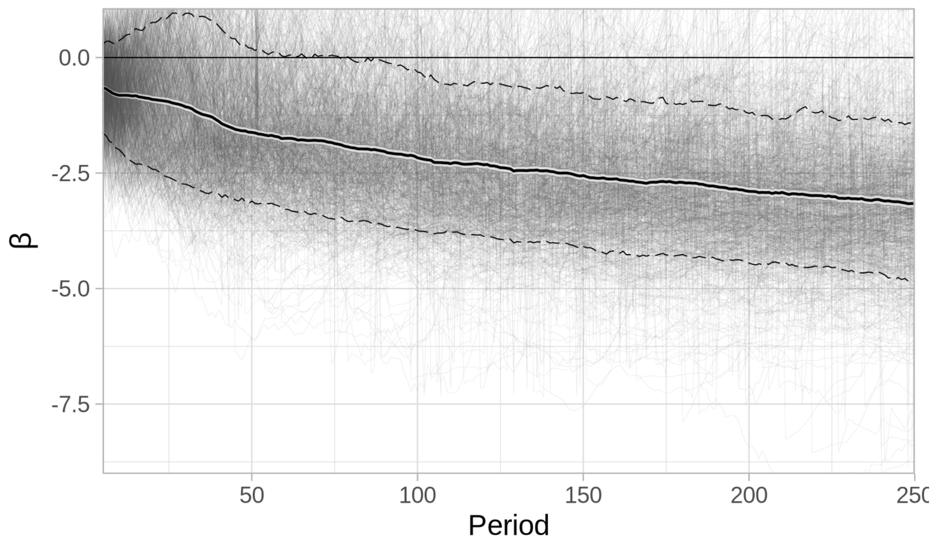


Figure 3: Effect of decreasing grade of membership, boundedly rational entry rule. $W = \alpha + \beta \cdot NGOM$

position. Consumers are homogeneously distributed along the line and choose among producers with preferences in line with the multinomial logit model (McFadden, 1974, 1984; de Palma et al., 1985; De Palma et al., 1987): A consumer located at position x considers the producer's quality, A_i, the distance to the producer, $|x - x_i|$, and idiosyncratic factors ϵ. Consumers choose between among existing producers and a ubiquitous residual product of zero quality. The market share of producer i among consumers located at x is given by:

$$s_i(x) = \frac{\exp(A_i - |x - x_i|)}{1 + \sum_j \exp(A_j - |x - x_j|)} \tag{1}$$

The total market share of producer i is the integral of market shares across all consumers in the market:

$$s_i = \int_{-\infty}^{\infty} s_i(x) dx$$

In practice, this value is easiest to calculate by intervals. In addition, since market share will be used to identify the optimal entry position for some focal producer, it helps to pin down the location of this focal producer as x_f and define various quantities with respect to that point. Define I_i as the interval between x_i and x_{i+1}; I_n is the open interval bounded on the left by x_n and I_0 is the open interval bounded on the right by x_1. It is also helpful to redefine producer i's quality on an interval I_j by $A_{ij} = A_i - |x_i - x_j|$, and to index consumer preferences by their distance from the endpoint closer to x_f: $d = x - x_i$ for intervals to the right of x_f, or $d = x_{i+1} - x$ for intervals to the left. With this redefinition of distance, it is also important to separately track producers depending on whether or not their appeal is increasing or decreasing in d: For intervals to the right of x_f for instance (i.e. for which $x_j \geq x_f$), producers located to the left of the interval (x_i s.t. $i \leq j$) decrease in appeal across the interval, and those to the right of it (x_i s.t. $i > j$) increase. With this in place, market share at a point (as given by expression (1)) for the producer at x_f (to the left of interval I_j) can be redefined as:

$$
\begin{aligned}
s_{fj}(d) &= \frac{\exp(A_{fj} - d)}{1 + \sum_{k \leq j} \exp(A_{kj} - d) + \sum_{k>j} \exp(A_{kj} + d)} \\
&= \frac{\exp(-d) \exp A_{fj}}{1 + \exp(-d) \sum_{k \leq j} \exp A_{kj} + \exp(d) \sum_{k>j} \exp A_{kj}} \\
&= \frac{\exp(-d) \exp A_{fj}}{1 + \exp(-d) K_l + \exp(d) K_r}
\end{aligned}
\tag{2}
$$

Producers whose value increases with distance do not exist on the open intervals I_0 and I_n (i.e. $K_r = 0$).

Integrating expression (2) on an open interval I_j gives combined market share of

$$
S_{fj} = \frac{\exp A_{fj}}{K_l} \log(1 + K_l) \tag{3}
$$

On an interior interval I_j of length $D_j = x_{j+1} - x_j$, combined market share is

$$
\begin{aligned}
\tilde{S}_{fj} &= \frac{1}{2}\left(D_j + \log(1 + K_l + K_r) - \log(1 + \exp(-D_j)K_l + \exp(D_j)K_r)\right) \\
R_{fj} &= \frac{\exp D_j - 1}{1 + 2K_l + (1 + 2K_r)\exp D_j} \\
S_{fj} &= \frac{\exp A_{fj}}{K_l} \cdot \begin{cases} \tilde{S}_{fj} + \frac{\tanh^{-1}\left(\sqrt{1-4K_lK_r}R_{fj}\right)}{\sqrt{1-4K_lK_r}}, & 4K_lK_r - 1 < 0 \\ \tilde{S}_{fj} + R_{fj}, & 4K_lK_r - 1 = 0 \\ \tilde{S}_{fj} + \frac{\tan^{-1}\left(\sqrt{4K_lK_r-1}R_{fj}\right)}{\sqrt{4K_lK_r-1}}, & 4K_lK_r - 1 > 0 \end{cases}
\end{aligned} \tag{4}
$$

As the length of an interval increases ($D_j \to \infty$), it can be shown that (4) converges to (3). Total market share for the producer at x_f is the sum of market share on all intervals:

$$
s_f = \sum_{j=0}^{n} S_{fj} \tag{5}
$$

In order to calculate the optimal entry position for a new entrant into this competitive landscape, it is again necessary to apply some measure of risk aversion. The reason why can be understood by inspection of (3): this value represents half the market share captured by a producer of appeal A_f. For low values of A_f, (3) is bounded below by 0; for high values, the expression becomes linear in A_f. In effect, this multinomial logit rule produces substantial option value for producers that are able to maximize the variance of A_f. On a Brownian landscape, producers will seek to maximize their distance from known positions, seeking to enter at infinite distance from prior competitors on an unbounded landscape. This tendency can be averted by imposing an increasing cost on low-appeal positions, or equivalently, by identifying utility function that imposes sufficient risk aversion. The minimum such function is the inverse of market share of an isolated producer:

$$
u(S) = \log(\exp(S/2) - 1)
$$

The value of entering at some position x_F is then given by integrating the expected utility of total market share across the distribution of appeal A_F at x_F. A_F follows a normal distribution with mean and variance given by the specific position on the Brownian walk. Finally, optimal entry can be found by maximizing expected utility across possible values of x_F.

Unlike the model described in the article, optimal entry positions can include the positions of incumbents, so that specific points on the landscape can become increasingly crowded over time. Such situations do not persist indefinitely, as the desirability of such positions decreases in the number of colocated competitors, until entering at open intervals (or other parts of the terrain) becomes more desirable.

The major downsides of this model, however, are (1) that calculation of the components of (4) requires high-precision arithmetic; (2) the lack of an obvious closed form integral for expected utility; (3) the requirement to numerically optimize the numerically derived expected utility. Barring substantial work to optimize these calculations, it is difficult to simulate these markets even to 100 periods.

Nevertheless, it is possible to simulate them to a limited extent. Fig. 4 reproduces Fig. 4 from the body of the article in markets using multinomial logit competition, showing the results from 100 markets run over 75 periods. As the figure suggests, the primary results of the paper appear to persist under this alternate definition of competition. In general, the results of the paper appear to be robust to various alterations of the basic competitive setup on a rugged Brownian landscape.

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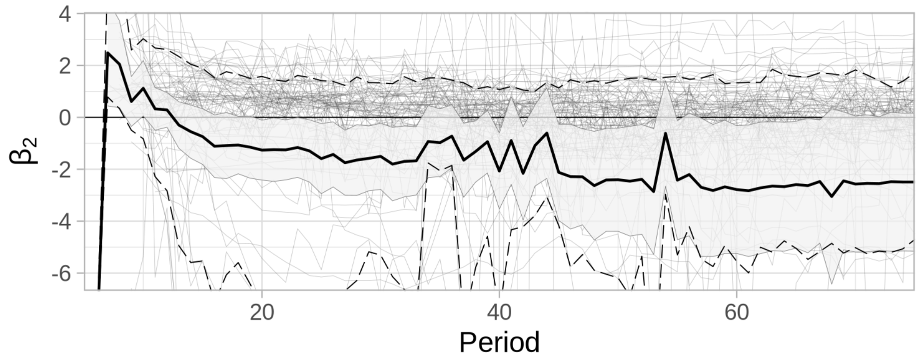
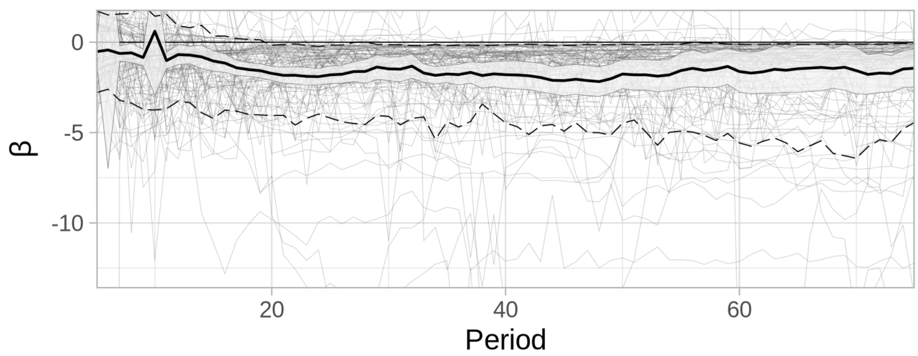
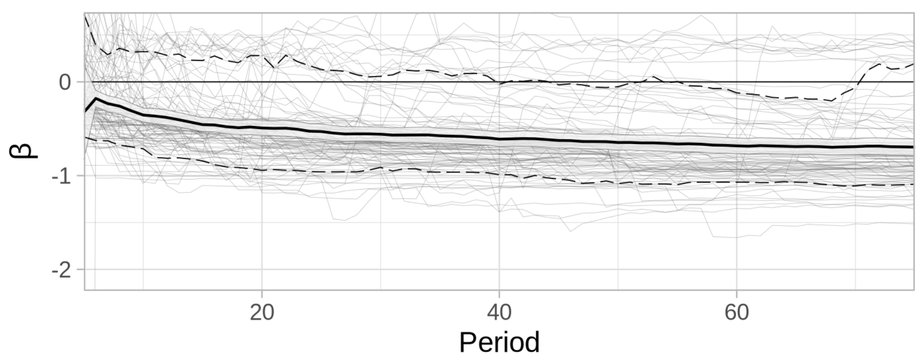


(a) Effect of category spanning. $W = \alpha + \beta_1 \cdot NoCategory + \beta_2 \cdot Spanner$



(b) Effect of decreasing grade of membership. $W = \alpha + \beta \cdot NGOM$



(c) Effect of increasing distance to neighbor. $W = \alpha + \beta \cdot distance$

Figure 4: Effect of position characteristics on position appeal, multinomial logit product choice, 100 simulated markets, 75 periods.